

Nuclear matter EOS with a three-body force

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Abstract

The effect of a microscopic three-body force on the saturation properties of nuclear matter is studied within the Brueckner-Hartree-Fock approach. The calculations show a decisive improvement of the saturation density along with an overall agreement with the empirical saturation point. With the three-body force the symmetry energy turns more rapidly increasing with density, which allows for the direct URCA process to occur in β -stable neutron star matter. The influence of the three-body force on the nuclear mean field does not diminish the role of the ground state correlations.

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1 Introduction

The theoretical prediction of the nuclear equation of state (EOS) is of great interest for understanding extreme states of matter as to density, temperature and isospin. In spite of the great deal of effort already done, yet this topic is largely controversial in many aspects, including the mechanism of nuclear saturation.

In non-relativistic approaches the model of nucleons interacting only via a two-body force fails to reproduce the empirical saturation observables. Thus phenomenological three-body forces (TBF), derived by the Urbana group, have been introduced with few adjustable parameters within both the variational method [1, 2, 3, 4] and the Brueckner theory [5, 6].

At the contrary, the success of Dirac-Brueckner calculations without TBF [7, 8, 9] seems to indicate that the mechanism of saturation is a purely relativistic effect. This effect can be traced to the virtual excitations of nucleon-antinucleon pairs and their contribution alone, estimated in Ref. [10], seems to be able to reproduce the correct saturation properties [11].

This raises the question of the role played by other important elementary processes such as the nucleonic excitations (Δ or Roper resonance). This point was examined already a decade ago by Grange et al. [12], who proposed a microscopic TBF based on the meson-exchange model. In that paper they reported a decisive improvement of both the saturation energy and density with respect to the existing Brueckner-Hartree-Fock (BHF) predictions based on pure two-body interactions [13]. Later more refined BHF calculations with two-body force gave results so sharply different [14] to call for a numerical re-examination of the BHF approach with the microscopic TBF.

In the present note this re-examination is made and new results for the EOS of nuclear matter are reported which benefit from a more sophisticated version of the two-body realistic interaction, i.e., the Argonne *AV18* [15]. The calculations are also extended to neutron matter with the purpose of extracting the symmetry energy which over the last years has been raising a strong interest for its remarkable astrophysical implications [16].

2 Effective three-body force

Our microscopic three-body force is deduced from the meson-exchange current approach. It contains the contribution due to the medium modification of the two-meson ($\pi\pi, \pi\rho, \rho\rho$) exchange part of the nucleon-nucleon (NN) interaction, the contribution associated to the σ and ω meson exchange and, finally, the $\rho\pi\gamma$ diagram. Their bare mass and free coupling constant are assigned to all mesons except the σ meson, for which a strong mass renormalization is expected as medium polarization effect. Moreover its value cannot be

completely dissociated from the other parameters giving the two-body force due to the self-consistent nature of the present approach [17]. The value of 540MeV has been adopted with the Paris force [12], but it is also compatible with the *AV18*, which is used in the present calculations. For a more detailed description of the model and approximations we refer to Ref. [12].

The effect of the TBF has been included in the calculation along the same line as in [12], where it is conveniently reduced to an effective two-body interaction to avoid the difficulty of the full three-body problem. A detailed description and justification of the procedure is given in [12]. Here we simply write down the equivalent two-body potential, which is given in r -space by

$$\begin{aligned} \langle \vec{r}_1 \vec{r}_2 | V_3 | \vec{r}_1' \vec{r}_2' \rangle &= \frac{1}{4} Tr \sum_n \int d\vec{r}_3 d\vec{r}_3' \phi_n^*(\vec{r}_3) [1 - \eta(r_{13})][1 - \eta(r_{23})] \\ &\times W_3(\vec{r}_1' \vec{r}_2' \vec{r}_3' | \vec{r}_1 \vec{r}_2 \vec{r}_3) \phi_n(r_3) [1 - \eta(r_{13})][1 - \eta(r_{23})] \end{aligned} \quad (1)$$

where the trace is taken with respect to the spin and isospin of the nucleon 3. The function $\eta(r)$ is the average over spin and momenta in the Fermi sea of the defect function, in which only the most important partial wave components have been included, i.e., the 1S_0 and 3S_1 partial waves. According to Eq. (1) the effective two-body interaction is obtained by averaging the three-body force over the wave function of the third particle taking into account the correlations between this particle and the two others.

As above mentioned the mass of σ meson is taken as a free parameter in the present TBF. This is due to the lack of information on the medium polarization effect on the σ meson propagator. The EOS could offer a chance of investigating this effect as far as the two-body force is also built on a meson-exchange model. For a numerical comparison with the results Ref. [12], we decided to adopt the same value of the σ -meson mass, i.e., $m_\sigma = 540 \text{ MeV}$, which anyhow makes the saturating effect of the TBF to be the most efficient.

3 Numerical results

Due to its dependence on the defect function the effective two-body interaction, Eq. (1), is calculated selfconsistently at each step of the iterative BHF procedure. Otherwise the procedure is the same as in standard BHF approach. The G -matrix is calculated selfconsistently along with the auxiliary potential by solving the Bethe-Goldstone equation. The continuous choice is adopted for the auxiliary potential because it provides a convergence of the hole-line expansion much faster than the gap choice [18]. The Argonne *AV18* is used for the two body nuclear force as it gives an excellent fit to NN scattering data as well as to the deuteron binding energy [15].

3.1 EOS in nuclear and neutron matter

In Fig. 1 the energy per nucleon is reported for both nuclear matter (lower curves) and neutron matter (upper curves). The effect of the three-body force can be assessed by the comparison between the BHF results with (solid curve) and without (short dashed curve) three-body force. The three-body force affects the EOS in such a way that in the low-density domain the energy per nucleon is practically unchanged, but in the high-density domain it rises up significantly. One main result for the EOS of symmetric nuclear matter is that the saturation density turns decisively improved towards the empirical value, from $0.265 fm^{-3}$ to $0.19 fm^{-3}$. At the same time there is no appreciable change in the saturation energy. This is a quite desirable feature of the three-body force, since the result for the saturation energy in our BHF calculation based on purely two-body interaction is already in good agreement with its empirical counterpart. All that makes us more confident with the general guess on the importance of the three-body force for the saturation problem of nuclear force. Compared to the old calculation with Paris force in Ref. [12] the new saturation density obtained in the present work is smaller and closer to the empirical value. This is not a surprise because in the calculation of Ref. [12] the saturation density predicted with only a two-body force was a bit larger than $0.3 fm^{-3}$.

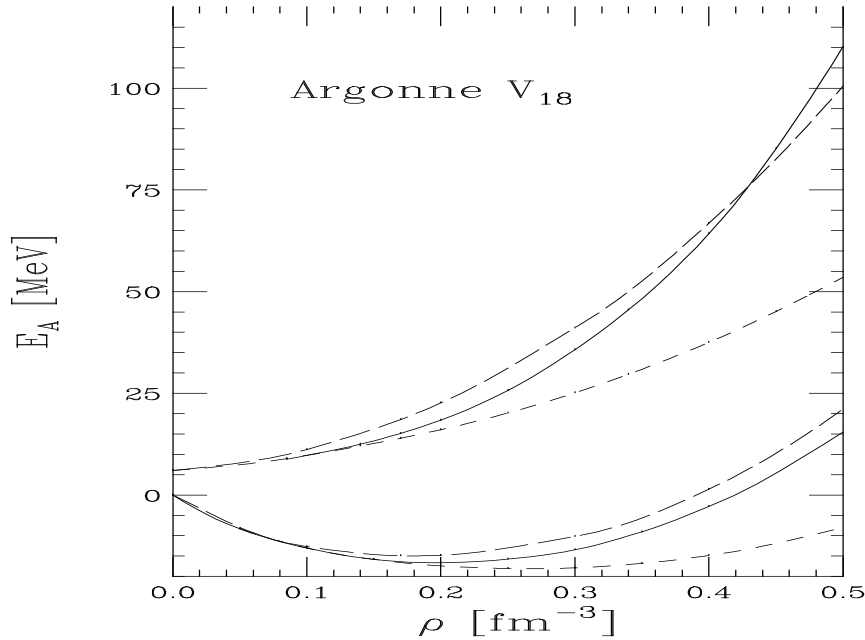


Figure 1: EOS of nuclear matter (lower curves) and neutron matter (upper curves). The Brueckner calculations are plotted for the AV18 without TBF (short-dashed lines) and with the TBF (solid lines) discussed in the text. For comparison are also reported the calculations with AV18 and Urbana TBF of Ref. [6] (long-dashed lines).

In Fig. 1 is also plotted the EOS's obtained from BHF calculations with two-body *AV18* force and phenomenological Urbana TBF [6] (longdashes). Despite the two different TBF's the two EOS's do not significantly differ from each other both for nuclear matter and for neutron matter. The Urbana TBF has two free parameters which were adjusted [5] in order to reproduce the saturation point, as shown in the latter EOS.

In recent variational calculations, which adopt the *AV18* combined with the recent Urbana *IX* TBF [3, 4], the two free parameters of the TBF are fixed to reproduce the binding energy of 3H and the saturation density of nuclear matter. But the resulting equilibrium energy of about $-12MeV$ for symmetric nuclear matter disagrees with the empirical value and no significant improvement is achieved by including relativistic boost corrections [4]. This discrepancy casts some doubt on constraining the TBF by means of the binding energy of light nuclei.

No less important is the effect on the stiffness of the EOS measured by the compressibility modulus

$$K = 9\rho^2 \frac{\partial^2 E_A}{\partial \rho^2} \quad (2)$$

calculated at the saturation density. The effect of three-body force on the compressibility is to enhance its value from 250MeV to 265MeV. This small change is due to the fact that the more pronounced curvature of the saturation curve is balanced by the smaller value of the saturation density.

The neutron matter EOS combined with that of symmetric nuclear matter provides us information on the isospin effects [16], in particular on the symmetry energy. The symmetry energy is defined as

$$E_{sym}(\rho) = \frac{1}{2} \left[\frac{\partial^2 E_A(\rho, \beta)}{\partial \beta^2} \right]_{\beta=0}. \quad (3)$$

It is well established [19] that the binding energy per nucleon E_A fulfills the simple β^2 -law not only for $\beta \ll 1$ as assumed in the empirical nuclear mass formula [20], but also in the whole asymmetry range. This enables us to calculate the symmetry energy E_{sym} in terms of the difference between the binding energy of pure neutron matter $E_A(\rho, 1)$ and that of symmetric nuclear matter $E_A(\rho, 0)$, i.e.,

$$E_{sym}(\rho) = E_A(\rho, 1) - E_A(\rho, 0), \quad (4)$$

but one would refrain from applying it at very high density. The results of our calculation for the symmetry energy as a function of baryonic density are depicted in Fig. 2.

As expected from the strongly repulsive TBF component at high density, a considerable enhancement of E_{sym} is found (compare the two solid lines). At the saturation point the enhancement is rather small, i.e., from 30.3 to 31.3MeV, which anyway is in agreement

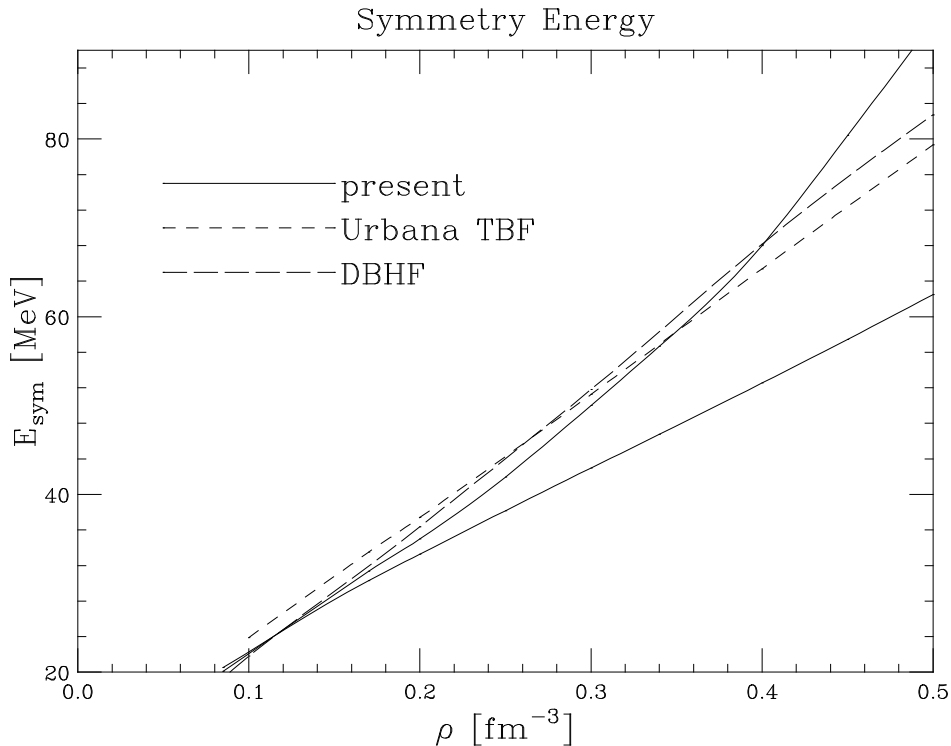


Figure 2: Density dependence of the symmetry energy in different approaches. The two solid lines correspond to the present approach without TBF (lower curve) and with TBF (upper curve), the short dashed curve to the EOS's with Urbana TBF (Ref. [6]) and the long dashed curve to a DBHF calculation with the Bonn one-boson-exchange potential (Ref. [9]).

with the empirical value of $30 \pm 4 \text{ MeV}$ [20]. Also reported are the symmetry energy obtained from the Urbana TBF [6] and the DBHF calculation [9]. Apart from the density range above 0.4 fm^{-3} , all three curves exhibit an almost linear increase of $E_{sym}(\rho)$ as a function of ρ and also a quantitative agreement with one another. A rather good agreement with the variational approach is also found extracting the symmetry energy from the EOS's reported in Refs. [3, 4]. The difference, which for instance at $\rho = 0.4 \text{ fm}^{-3}$ is a bit less than 15%, would be attributed to the fact that the variational calculation underbinds symmetric nuclear matter.

The close similarity of our result with the DBHF prediction is quite astonishing if one considers the two different contexts, i.e. the relativistic DBHF without a TBF in Ref. [9] and the present non-relativistic BHF with TBF, in which they have been obtained. On the other hand, we already mentioned the equivalence between the medium effects on the Dirac spinor in the Dirac-Brueckner approach and the virtual excitation of $N\bar{N}$ pairs, which is an important component of the present TBF [11]. In spite of the good agreement in E_{sym} , one expects significant differences for observables which are very sensitive to E_{sym} such as the proton fraction $Y = \frac{Z}{A}$ in β -equilibrium nuclear matter, which approximately

fulfills a power law, i.e., $Y \simeq E_{sym}^3$ [21]. Due to the important implications for the cooling mechanism in neutron stars [22], a comparison among the different predictions is worth to be done. The proton fraction is calculated from the equilibrium condition in nuclear matter formed by mixture of neutrons, protons, electrons and muons with respect to weak interaction. Assuming charge neutrality, one easily gets

$$3\pi^2(\hbar c)^3 \rho Y = [4E_{sym}(1 - 2Y)]^3 + \{[4E_{sym}(1 - 2Y)]^2 - m_\mu c^2\}^{3/2} \theta(\mu_e - m_\mu c^2) \quad (5)$$

where the step function $\theta(\mu_e - m_\mu c^2)$ indicates that when the electron chemical potential exceeds the muon rest mass, the muon channel opens and muons participate to the chemical equilibrium. Within this context we calculated the proton fractions corresponding to the different E_{sym} reported in Fig. 2. The values of $Y(\rho)$ are reported in Tab. I. It has been shown [22] that the direct URCA process can occur if the proton fraction exceeds certain threshold values of Y , which has been estimated of about 0.148 in the case of $\mu_e \gg m_\mu c^2$. This value serves as a good approximation for a simple estimate. For example, at the density $\rho = 0.45$, the critical value of Y is about 0.136 for electrons and about 0.161 for muons. It is seen that in the density domain here considered (up to 0.5 fm^{-3}) this value is not reached in the non-relativistic calculation based on purely two-body force, whereas it is reached in the range $0.4 - 0.5 \text{ fm}^{-3}$ in the other cases. Comparing the data of last column with the data of Ref. [6], where the *AV14* is used as two-body potential, we notice that the *AV18* interaction has the effect of reducing the density threshold from 0.5 fm^{-3} to a value between 0.4 and 0.5 fm^{-3} .

Tab. I Proton fractions vs density for β -equilibrium nuclear matter: our results from Argonne V18 without (second column) and with TBF (third column), Dirac-Brueckner calculations from Ref. [9] (forth column) and and BHF from *AV18*+*Urbana* from Ref. [6] (last column).

ρ	BHF	BHF+TBF	DBHF	BHF+Urbana
0.04	0.0161	0.0138		
0.06	0.0210	0.0181		
0.085	0.0245	0.0229		
0.10	0.0264	0.0264	0.0241	0.031
0.14	0.0328	0.0338		
0.17	0.0378	0.0417	0.0403	0.050
0.20	0.0440	0.0508	0.0528	0.060
0.30	0.0659	0.0920	0.0993	0.096
0.40	0.0865	0.1365	0.1364	0.128
0.50	0.1069	0.1882	0.1616	0.155

Before we conclude, let us give a brief discussion about the single-particle (s.p.) potential. It is found that at low density ($\rho \leq \rho_0 = 0.17\text{fm}^{-3}$), the effect of the TBF on the s.p. potential is rather small. At the saturation density $\rho_0 = 0.17\text{fm}^{-3}$, only a small increase about $1.0 \sim 1.5\text{MeV}$ in symmetric nuclear matter is observed at the momentum around the Fermi momentum k_F , which amounts a reduction of the attractive mean field of less than 2%. This is consistent with the result for the binding energy E_A as discussed before and suggests that it is still necessary, even after introducing the TBF, to include the ground state particle-hole excitations (the second-order M_2 contributions) in the mass operator in order to get a satisfactory agreement to the phenomenological optical potential [16, 23]. As the density increases, the repulsive contribution of the TBF to the s.p. potential becomes increasingly pronounced since the TBF plays its major role at high density. For example, at $\rho = 0.34\text{fm}^{-3}$ for symmetric nuclear matter, there is an enhancement over the whole momentum range (from -130.8 to -123.7MeV at $k = 0$ and -94.4 to -80.3MeV at k_F). Again this is in agreement with the observation on the binding energy E_A .

4 Summary and conclusions

In summary, we have investigated the effect of a microscopic three-body force on the EOS of symmetric nuclear matter and pure neutron matter within the Brueckner approach. The introduction of a TBF turns out to be crucial for reproducing the empirical saturation point, confirming the previous investigations with phenomenological TBF. But the microscopic TBF has the advantage of tracing the properties of nuclear matter back to more fundamental degrees of freedom. In addition it is suitable for a closer contact with relativistic approaches.

In particular, the saturation density approaches closely the empirical value after including the TBF in the calculation. At the contrary the saturation energy turns out to be almost unaffected which is a desirable result since, in the present calculation, its value only with two-body force is already in good agreement with the empirical binding energy. Accordingly, the TBF modifies appreciably the mean field only above the saturation density, so confirming the limit of BHF approximation without ground state correlations in describing the phenomenological optical potential. From the calculation of the EOS for pure neutron matter we have extracted the symmetry energy, which turns in good agreement with the relativistic DBHF calculations and also with the BHF and variational calculations adopting phenomenological Urbana TBF.

The steep uprise of the energy symmetry with density could have deep influence on the properties of neutron stars and on the supernovae explosions. The proton fraction

has been calculated under conditions of β -equilibrium. The threshold value necessary to switch on the direct URCA processes is reached below 0.5fm^{-3} when the three-body force is included. This result may have remarkable consequences for the neutron-star cooling mechanism.

Finally, we address the question whether or not the EOS of nuclear matter can probe the medium modifications of the NN interaction. Our approach to the TBF looks suitable for such a purpose provided that the two-body interaction is also described in terms of a meson-exchange model.

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